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THEORETICAL BACKGROUNDS FOR ENHANCEMENT OF DRY MILK DISSOLUTION PROCESS: MATHEMATICAL MODELING OF THE SYSTEM "SOLID PARTICLES – LIQUID"

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Abstract: The mathematical model of immersion of insoluble spherical particle with smooth surface under absolute statics (incl. assumption – its spontaneous formation on the surface) at the particle density ranging from 1.05 to 1.75 kg/m^3 and contact angle of moistening from 0º to 180º was created for development of theoretical and practical backgrounds of the reconstitution process. This model was used as the base of model of immersion in water and drowning of cubic grid of spherical insoluble particles under full static condition. Regularities of layers' drawning were established and an algorithm for calculating the missing force for full grid immersion was developed. It is possible to determine the coefficient of correlation between the calculated and actual data, taking into account the heat and mass transfer processes occurring during the dissolution of the dry products that will bring model to real systems and, in such a way, unify the process.

Keywords: reconstitution, mathematical model, dry milk products, a particle, the particle frame

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INTRODUCTION

It is known that basic volumes of dry milk products (DMP) processing in different branches of the industry suppose the availability of the tentative process of their solution in water. Practical results show that technical-technological presentation of the mentioned process stipulates to a great extent the qualitative characteristics and quantitative yield of the product, efficiency of the technological equipment operation and impacts indirectly the enterprise working routine [1].

A wide assortment and heterogenous nature of DMP raw material, disparity in operating principles of the industrial equipment as well as undeveloped methodological base of evaluative criteria of the process efficiency make it impossible to find dependence by generalization of production and experimental data. Accordingly the actual task is substantiation of the rational parameters of reconstitution technology from the point of the process simulation with the model complication from monoparticle immersion with variable form, density range and wetting contact angle in static conditions up to real polycomponent system in dynamics.

METHODOLOGY OF INVESTIGATION

The object of investigation – the mathematical model of immersion of insoluble spherical particle with smooth surface under absolute statics (incl. assumption – its spontaneous formation on the surface) at the particle density from 1050 to 1750 kg/m^3 and contact angle of moistening from 0°C to 180°C. The liquid with physical-chemical parameters of distilled water [2, 3] was used as solvent component of the system. The model should be able to determine the particle optimal minimal weight under the condition of its sphericity, required for overcoming surface tension. It is necessary as well to set up the rate of immersion after its sinking if: the particle is insoluble, the pass length is endless. Thus interaction of the particle with the liquid is considered in two variants: solid – liquid – gas (S/L/G) if the contact is running on the liquid surface and liquid – solid (L/S) when the particle is completely immersed. It is evident that force ratio influencing the particle is not always constant and either sets up the balance in the frame of the defined variant or determines its transition from one to another.

Methods of investigation – adaptive integration of hydrostatics and hydrodynamics classical results into applied fields of technology.

1. Critical Parameters of the statistic systems for immersion

The scheme of balanced interaction at S-L-G three-phase contact is presented at Fig. 1 (projection at Oxz). Fig. 2 shows visualization of wetting angle virtual configuration. The balance supposes balancing of all forces effecting the particle: Buyancy forces, surface tension forces and gravity forces.

We have the following system parameters: R – particle radius, m; ρ – particle density, kg/m^3 (values range 1100–1700); θ – angle of wetting, ^{o} (values range – 0–180); ϑ – angle of immersion, ^{o} (values range considering density of the studied particle 0–90), the particle embedding occurs at $\vartheta = 0$ (considering casual fluctuations near water surface the critical values for spontaneous embedding $\vartheta_c = 5^o$ are accepted); ρ_0 – liquid density (water), kg/m^3 (assumed equal to 1000; σ – surface tension, H/m (assumed equal to $72{,}86 \cdot 10^{-3}$); g – acceleration of gravity, m/s^2 (assumed equal to 9.8) [1, 4–8].

Introduce the symbol (difference between angle of wetting and angle of immersion is the basic parameter influencing the system balance, see then):

$$\alpha = \theta - \vartheta. \tag{1}$$

1.1. Surface tension force

Definition 1. Limiting wetting angle θ – the angle at the point of three-phase contact between tangents to particle surface and to liquid surface.

Surface tension force forms the angle at contact point with tangent to the particle equal to wetting angle. Modulus of vertical projection at surface tension axis Oz equals to:

$$\sigma_z = \sigma \cos\left(\frac{\pi}{2} - \theta + \vartheta\right) = \sigma \sin\alpha\,. \tag{2}$$

Calculation of surface tension force is carried out by the length of three-phase interface. In this case it is radius circumference:

$$r = R\sin\vartheta. \tag{3}$$

Accordingly circumference is:

$$l = 2\pi R \sin\vartheta. \tag{4}$$

The resultant value of Oz force component is:

$$F_\sigma = 2\pi R\sigma \cdot \sin\vartheta \cdot \sin\alpha. \tag{5}$$

1.2. Radius of liquid curvature

Imagine that uncurved liquid surface contacted the particle at L point (Fig. 3) and rose to point A over particle surface by R radius, ϑ_o – angle between radiuses scored to points L and A.

Draw perpendicular through point A to liquid surface up to point B and then the height capillary rise equals the length of AB segment.

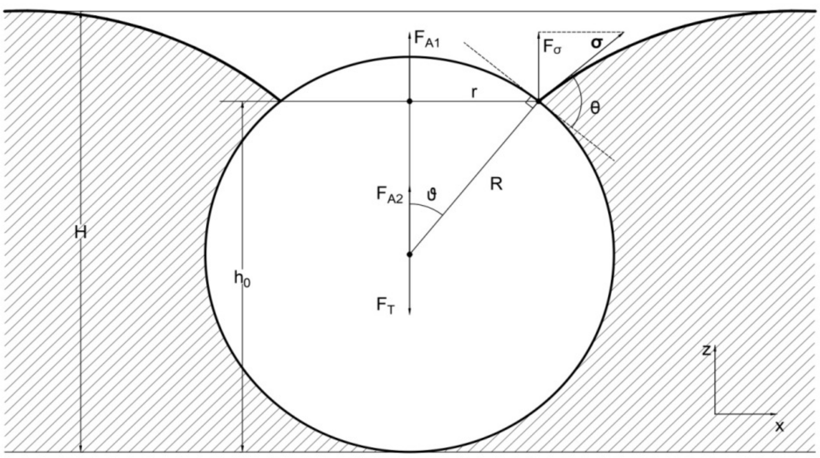
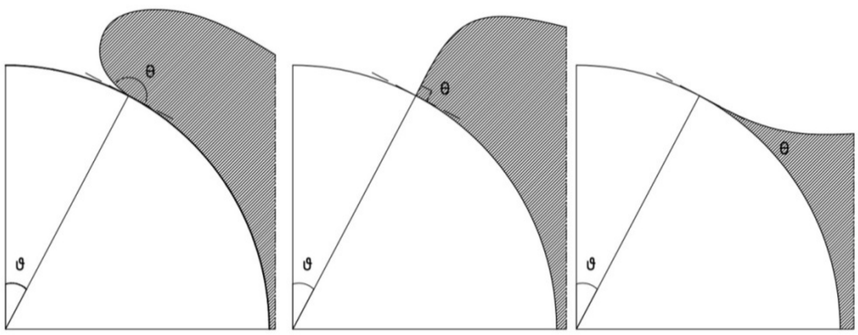


Fig. 1. Immersion of the particle into liquid.



(a) (b) (c)

Fig. 2. Visualization of wetting angle configuration: (a) $\theta = 180^o$, (b) $\theta = 90^o$, (c) $\theta = 0^o$.

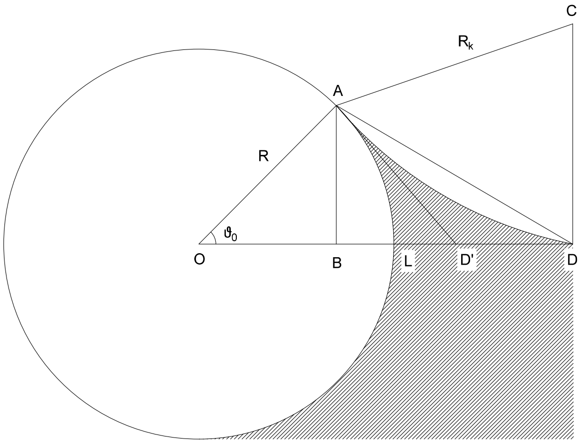


Fig. 3. The particle wetted by liquid.

The sizes of the studied particles are small then the amounts of clearances are relatively not big. The profile of curved liquid surface is spherical as the impact of gravity forces on it is practically low. Thus meniscus profile represents the circle touching the particle at contact point A and touching the surface of uncurved liquid at point D. So the circle lies at two tangents AD' and DD'. In triangle OAD' the angle OAD' equals the sum of the right angle and wetting angle.

$$OAD' = \frac{\pi}{2} + \theta, \quad (6)$$

$$AD'O = \frac{\pi}{2} - \alpha \cdot \quad (7)$$

Thus the angle between tangents equals

$$AD'D = \alpha\ . \quad (8)$$

To calculate the radius of curvature examine triangle AOD'. According to theorem of sines:

$$\frac{R}{\cos\alpha} = \frac{AD}{\sin\vartheta_0}\ . \quad (9)$$

Finally, the radius of curvature can be found from right triangle AD'C where angle AD'C equals α/2

$$AD' = \frac{\sin\vartheta_0}{\cos\alpha}\tan\frac{\alpha}{2}R\ . \quad (10)$$

Assuming the liquid radius of curvature we can calculate $h=|H-h_o|$ at which the liquid rises [9]:

$$h = |2\sqrt{\frac{\sigma}{\rho_0 g}}\sin\frac{\vartheta}{2}|. \quad (11)$$

1.3. Buoyancy force

Buoyance force is up-directed endwise O_z and is divided into two parts: F_{A1} – the force acting on the wetted but not immersed below the level of uncurved liquid surface and F_{A2} – the force acting on the immersed part of the particle.

The first component. If the immersed segment of the particle was replaced by liquid hydrostatic pressure would be equal to the weight of liquid column with by volume.

We have correlation for force:

$$F_{A1} = 2\pi r^2\sqrt{\rho_0 g\sigma}\sin\frac{\vartheta}{2}. \quad (12)$$

The second component of buoyancy force acts at the sphere segment with height

$$h_0 = R(1+\cos\vartheta). \quad (13)$$

Accordingly the segment volume equals to:

$$V = \int_0^{R\cos\vartheta}\pi\,(R^2 - x^2)\,dx +$$

$$+\int_0^R \pi\,(R^2 - x^2)\,dx = \frac{1}{3}\pi R^3(2-\cos\vartheta)(\cos\vartheta+1)^2. \quad (14)$$

As a result we have the equation for the second component of buoyancy force:

$$F_{A2} = \frac{1}{3}\pi R^3(2-\cos\vartheta)(\cos\vartheta+1)^2\cdot\rho_0\cdot g. \quad (15)$$

1.4. Gravity force

Gravity force is down-directed along axis O_z and equals to:

$$F_T = \frac{4}{3}\pi R^3\rho g. \quad (16)$$

1.5. Balance of forces

Buoyancy force and projection of surface tension force to axis O_z are directed upwards, gravity force is directed downwards. Considering that note the balance equation where the particle will be in equilibrium state in liquid:

$$F_\sigma + F_{A1} + F_{A2} - F_T = 0. \quad (17)$$

Fill incoming forces – equations (5), (11), (15), (16)

$$[(2-\cos\vartheta)(\cos\vartheta+1)^2\rho_0 g - 4\rho g]R^2 +$$

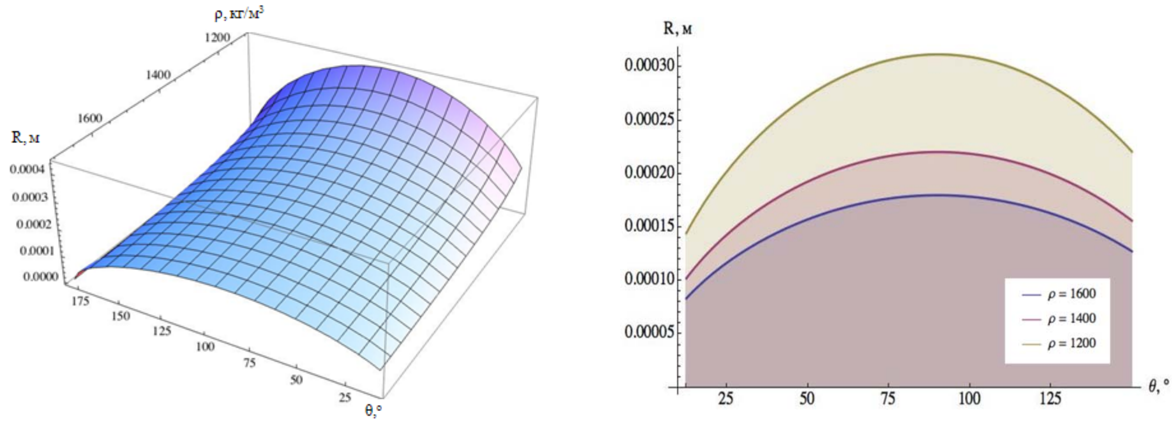
$$+6\sqrt{\rho_0 g\sigma}\sin^2\vartheta\,\sin\frac{\vartheta}{2}\,R + 6\sigma\sin\vartheta\sin\alpha = 0. \quad (18)$$

It is possible to specify for this system the particle radius required for determination balance of forces with angle of immersion. In this case spontaneous particle immersion at low values of (0–5°) occurs.

Dependence of R(p) is presented at Fig. 4 and 5. Minimal size of the particle required for spontaneous immersion equals to 0.05 mm for ϑ =5 (wetting angle $\vartheta = 5^o$, density – 1700 kg/m^3) and 0.15 mm for $\vartheta = 0.1^o$ (wetting angle $\vartheta = 5^o$, density – 1700 kg/m^3). At $\alpha \geq 0$ the equation has the solution as:

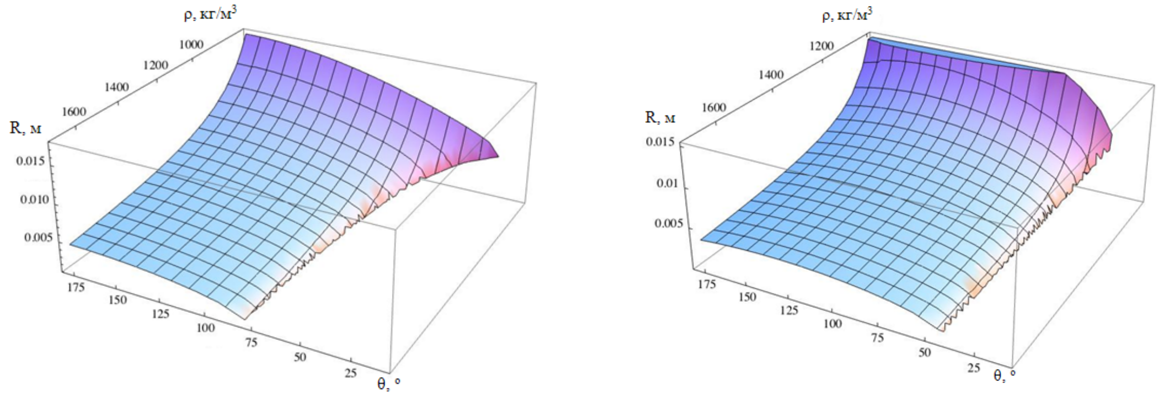
$$\rho_0 < \rho$$
$$(2 - \cos\vartheta)(1 + \cos\vartheta)^2 < 4$$

At $\alpha < 0$ two variants are possible: absence or presence of strictly one value of radius for each specific angle of immersion in the concerned range of the system parameters (ϑ and p). The example is shown at Fig. 6 – the particle location with p = 1400 kg/m^3 and value $\vartheta = 50^o$ is not stabilized at the angle $\vartheta = 90^o$ (a) under no radius, but is stabilized at $\vartheta = 45^o$ (b).



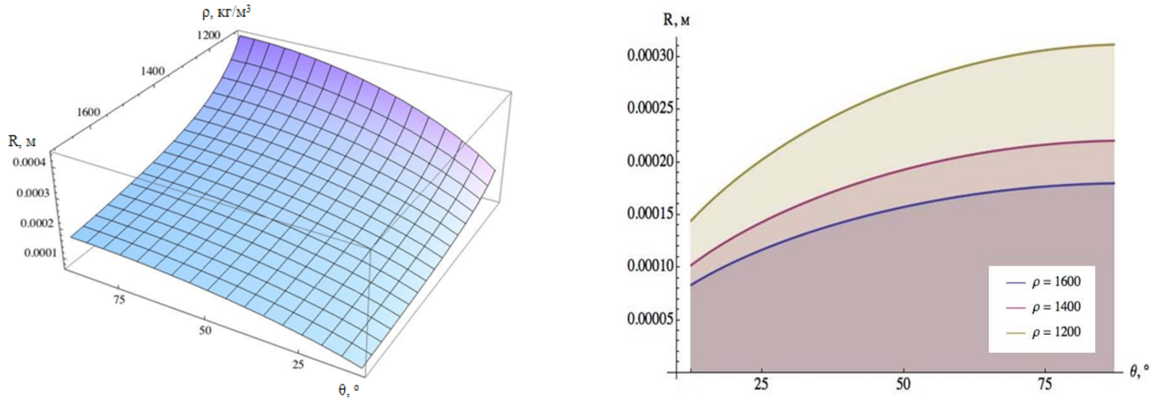
(a) (b)

Fig. 4. R (ϑ, p) – minimum radius at $\vartheta = 5^o$ (a) with detailing at fixed values of density (b).



(a) (b)

Fig. 5. R (ϑ, p) – minimum radius at $\vartheta = 0.1^o$ with detailing at fixed values of density.



(a) (b)

Fig. 6. R (ϑ,p) – minimum radius at $\vartheta = 90^o$ (a) and $\vartheta = 45^o$ (b).

2. Fluid motion

2.1. Behavior after sinking

After breakage of three-phase contact limit (at the excepted terms, with obtaining of critical angle of immersion ϑ) Buoyancy force working on the particle may be presented as the unified value. Therefore force F_α promoting the particle sinking (down-directed) is the difference of two forces:

$$F_a = \frac{4}{3}\pi g R^3 \cdot (\rho - \rho_0). \qquad (19)$$

As the particle is assumed as spherical and has small size Stokes low is accepted. Force F_d of water head resistance according to this law equals to:

$$F_d = 6\pi \nu v R, \qquad (20)$$

where ν is the liquid dynamic velocity, Па ·c, v is the particle velocity, m/s. Then the second Newton's law is acceptable for determination of embedding velocity:

$$F_a - F_d = \rho g a, \qquad (21)$$

where α is the particle acceleration, m/s^2 .

We have Cauchy problem relatively to the particle velocity v (at zero time moment velocity is supposed to be zero):

$$\begin{cases} \rho g \cdot \dot{v} + 6\pi \nu R \cdot v - \frac{4}{3}\pi g R^3 \cdot (\rho - \rho_0) = 0 \\ v(0) = 0 \end{cases} .(22)$$

The following function will be the solution:

$$v(t) = \frac{2}{9}\frac{gR^2(\rho-\rho_0)}{\nu}\Big(1 - \\ - \exp\Big\{-\frac{6\pi\nu R}{\rho g}t\Big\}\Big). \qquad (23)$$

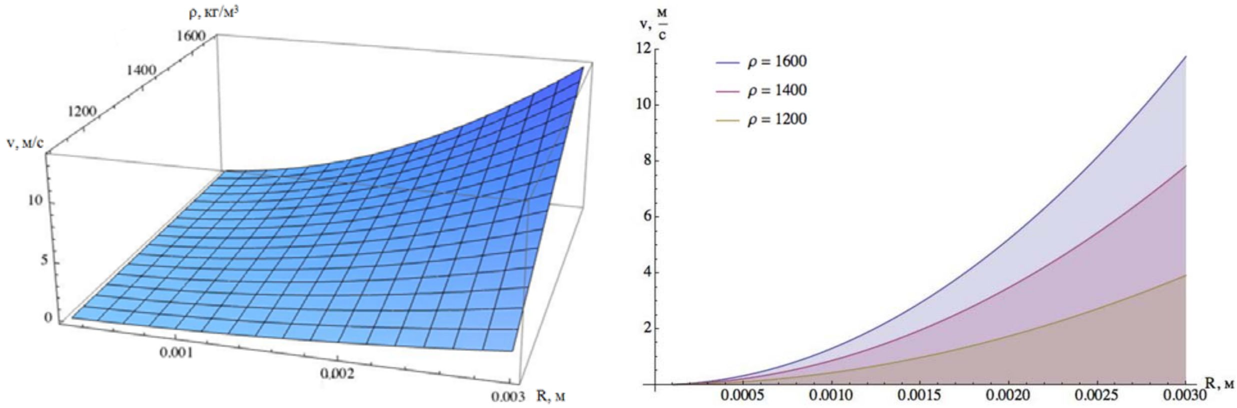
Accepting the value of viscosity dynamic coefficient equals to 0.001 Па · c (at 20^{o}C) we obtain the dependence of the fixed velocity and the particle density specified at Fig. 7.

2.2. Liquid motion in clearance

In this paragraph we present the theoretical base of capillary impregnation influencing sinking of powder milk products located on the liquid surface. The process of capillary impregnation in the simplest form represents filling of clearance between the particles. The liquid starts motion in the formed clearance due to pressure of the twisted liquid surface. We shall examine the liquid motion between adjacent particles.

As pressure at the upper mark of the lattice from spheres is permanent we can suppose that pressure change on the lattice height will be only the function of the liquid fluid velocity along this lattice (Fig. 8).

$$\frac{dP}{dz} = -f(u). \qquad (24)$$



(a) (b)

Fig. 7. v (R,p) – the particle fixed velocity after balance of forces subject to radius (a) with detailing at the fixed values of density (b).

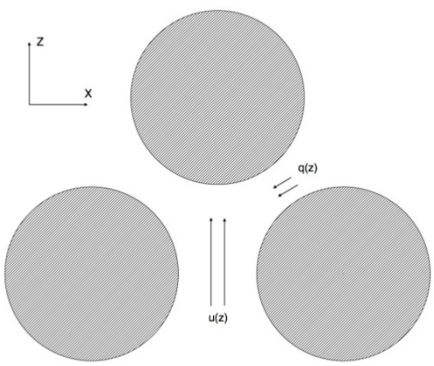


Fig. 8. Liquid motion in the clearance u(z) – motion velocity of the liquid along the lattice from spheres, m/s; dq(z)- liquid inflow from adjacent cells, m^3 /s.

With regard to geometrical shape it is advisable to calculate in co-ordinate cylindrical system.

For a start Gashen- Puazel formula can be used for viscous liquid flowing along the lattice body:

$$f(u) = \frac{8\mu u}{r_\varphi{}^2}, \qquad (25)$$

where μ is the dynamic coefficient of viscosity, Pa s; r_φ is the radius of lattice elementary tube, m.

The known Dupui formula can be used for determination of liquid amount entering at the dz height:

$$dq(z) = \frac{2\pi k_1 (P_z - P(z))dz}{\mu \ln\left(\frac{R}{r_\varphi}\right)}, \qquad (26)$$

where P_3 is the external pressure on the net, Pa; R is the radius of zone of liquid accumulation, m.

The liquid flowing from z mark will be:

$$dq = \pi r_\varphi{}^2 u(z). \qquad (27)$$

Thus liquid velocity at z+dz mark will be:

$$u(z+dz) = \frac{\pi r_\varphi^2 u(z) + dq(z)}{\pi r_\varphi^2} = u(z) + \frac{2k_1(P_z - P(z))dz}{\mu r_\varphi^2 \ln(\frac{R}{r_\varphi})}. \qquad (28)$$

Consequently, according to formula (26) we have the chance to forecast liquid velocity at every height

$$\frac{dP(z+dz)}{dz} = -f(u(z+dz)). \qquad (29)$$

Then taking into account the previous value change at section dz will be:

$$\frac{dP(z+dz)}{dz} - \frac{dP(z)}{dz} = -(f(u(z)) - f(u(z+dz)), \qquad (30)$$

$$\frac{d^2P(z)}{dz^2} = -\frac{2k_1(P_3 - P(z))dz}{\mu r_\varphi^2 \ln(\frac{R}{r_\varphi})} f'(u(z)). \qquad (31)$$

In equation (31) two functions P(z) and u(z) are unknown. According to (27) we can find u(z) using Lagrange theorems and values dz = 0:

$$u(z) = \frac{dq}{\pi r_\Phi^2} = \frac{2k_1(P_3 - P(z))dz}{r_\Phi^2 \mu \ln\left(\frac{R}{r_\Phi}\right)} = \frac{2k_1}{r_\Phi^2 \mu \ln\left(\frac{R}{r_\Phi}\right)} \int_0^z (P_3 - P(z))dz. \qquad (32)$$

Edge conditions for u(z) function are calculated by means of clearance curvature radius.

3. Build-in of the particle lattices

We shall examine the conditional cubic lattice broken to squares. Spherical particles of identical radius are located in lattice sites and at that the particle layers are located in staggered order (Fig. 9). The lattice is immersed into liquid with water characteristics transversely to the surface.

We shall examine the model in which M kg of dry milk product is filled into reservoir with water with surface area S m^2. It is required to calculate solution rate of the whole volume of dry milk product with the given impact force Fv and known parameters of particles R, p, θ, u.

We shall modify balance of forces for immersion of one particle described subject to radius of the liquid curvature between particles and capillary impregnation process.

$$F_\sigma + F_{A1} + F_{A2} - F_T + F_v = ma,$$
$$F_\sigma = 2\pi R\sigma \cdot \sin\vartheta \cdot \sin\alpha,$$
$$F_{A1} = 2\pi r^2 \sqrt{\rho_0 g \sigma} \sin\frac{\vartheta}{2}, \qquad (33)$$
$$F_{A2} = \frac{1}{3}\pi R^3 (2 - \cos\vartheta)(\cos\vartheta + 1)^2 \cdot \rho_0 \cdot g,$$
$$F_T = \frac{4}{3}\pi R^3 \rho g N.$$

Solving the obtained differential equation (7)

$$F(\vartheta(t)) = m\ddot{\vartheta}(t). \qquad (34)$$

Relatively to angle of immersion one can calculate the time required for immersion of one layer – obtaining (t) = 180°C.

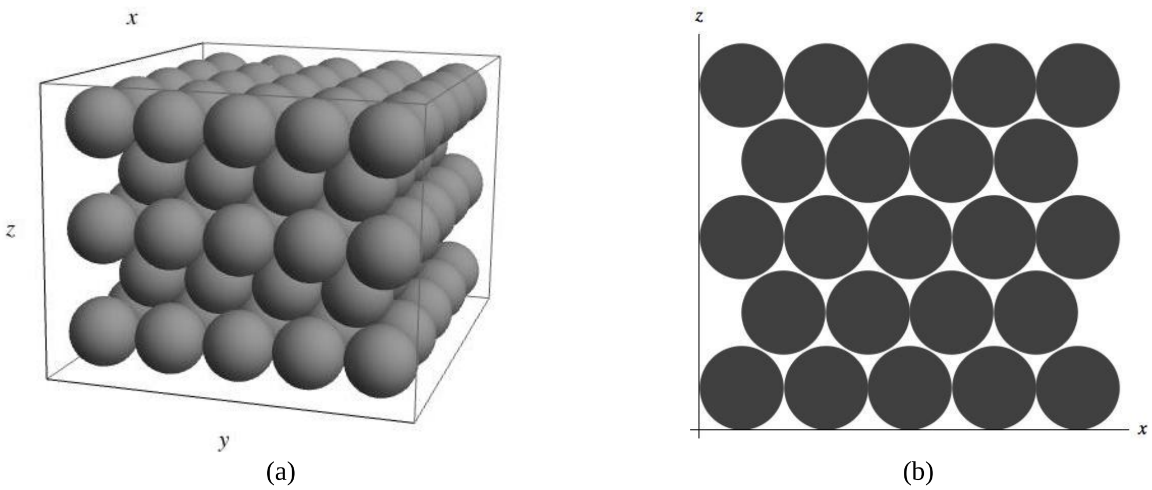
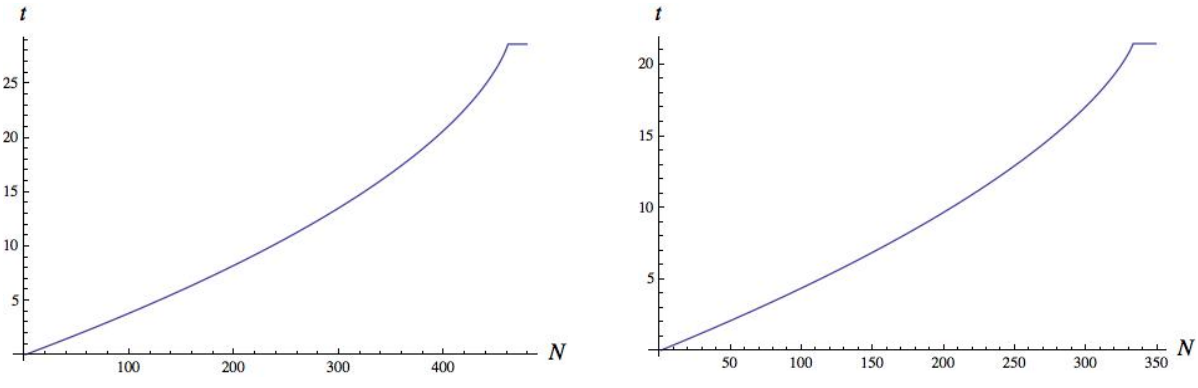


Fig. 9. Packing of the lattice from particles (a) and profile on surface Oxz (b).

Let us examine two DMP – dry whole milk (p = 1320 kg/m^3 and dry skim milk (p = 1510 kg/m^3). The sizes of these products are in the range of 0.05….0.25 mm; for manufacture of 1 t of the product 125 kg and 90 kg, respectively are required. Theoretical rates of solution in 1 m diameter tank is presented at Fig. 10. According to the drawing the layer portion will be remained on the liquid surface without application of force. Fig. 11 shows dry skimmed milk solution under secondary force action.



(a) (b)

Fig. 10. Period of dry milk product immersion (a) Dry whole milk (DWM), (b) Dry skim milk (DSM).

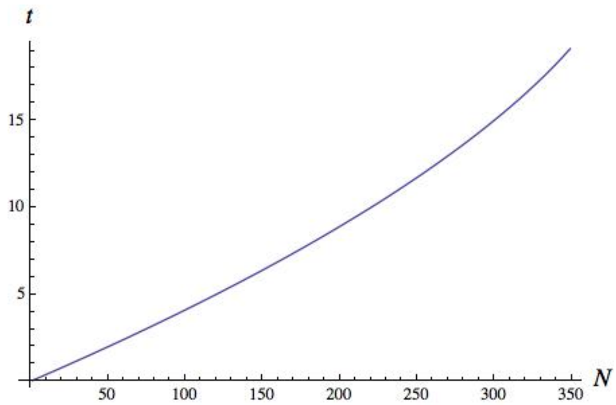


Fig. 11. Dry skim milk (DSM) solution under 4053 Pa pressure.

RESULTS

Simulated model of immersion in water and drowning of cubic grid of spherical insoluble particles under full static condition. Established regularities of layers' drawning and developed an algorithm for calculating the missing force for full grid immersion. In the future, it is possible through pilot studies to determine the coefficient of correlation between the calculated and actual data, taking into account the heat and mass transfer processes occurring during the dissolution of the dry products that will bring model to real systems and, in such a way, unify the process.

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